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Enriques surfaces covered by Jacobian Kummer surfaces

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1 Introduction

Jacobian Kummer surface X

$$
\begin{array}{c}
\text{C: a genus 2 curve} \\
\Downarrow \\
J(C) = \mathrm{Pic}^0(C)\text{: Jacobian of } C \\
\Downarrow \\
\overline{X} := \overline{Km}(J(C)) := J(C)/\{\pm 1\} \\
\textit{(Kummer's quartic surface)} \\
\Downarrow \\
X := Km(J(C))\text{: the minimal desing. of } \overline{X}.
\end{array}
$$

$$
\begin{array}{ccc}
J(C) & & \\
\downarrow{\scriptstyle /\pm 1} & & \\
\overline{X} & \xleftarrow[\text{min.desing.}]{} & X = Km(J(C))
\end{array}
$$

Definition
X is Picard-general if $\rho(X) = 17$, which we assume in what follows.

$\mathrm{Aut}(X)$ has been studied by many authors.
One definitive result is the following
Theorem(S. Kondo, 1998)
$\mathrm{Aut}(X)$ is generated by

$$
\begin{cases}
16 \times 4 & \text{Klein's involutions } (t_\alpha, \sigma_\beta, p_\alpha, p_\beta), \\
60 & \text{Hutchinson's involutions } (\sigma_G), \\
192 & \text{Keum's automorphisms } (\phi_{W,W'}).
\end{cases}
$$

Where

$$\alpha \in \{\text{2-torsion pts of } J(C)\},$$
$$\beta \in \{\text{theta characteristics of } C\}.$$

Corollary of the Main Theorem
$\mathrm{Aut}(X)$ is generated by

$$
\begin{cases}
16 \times 4 & \text{Klein's involutions } (t_\alpha, \sigma_\beta, p_\alpha, p_\beta), \\
60 & \text{Hutchinson's(HG) involutions } (\sigma_G), \\
192 & \underline{\textit{Hutchinson-Weber(HW) involutions}}\,(\sigma_W).
\end{cases}
$$

Where did σ_W come from ?

2 Main Result

Main Theorem There are $31 = 6 + 10 + 15$ fixed-point-free involutions on X, up to the isomorphism of the quotient Enriques surfaces.
They are exactly as follows.

3 free involutions on X

Switches

$$\Theta_\beta = \{p - \beta | p \in C\}.$$
$$\text{For } p \in J(C),\ (\Theta_\beta + p) \cap (\Theta_\beta - p) = \{q, -q\}.$$
$$\sigma_\beta\colon\ \pm p \mapsto \pm q.$$
$$\sigma_\beta \in \mathrm{Bir}(\overline{X}) = \mathrm{Aut}(X).$$

β runs over even theta characteristics of C; we obtain 10 free switches.

HG involutions

Restriction of the Cremona involution to $\overline{X}$:

$$\sigma_G : (x, y, z, t) \mapsto \left(\frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{t}\right).$$

G : four points of $\overline{X}$, called Göpel subgroup.

σ_G is well-defined, because
Theorem[Hutchinson] If we choose the four points of G as the reference points of $\mathbb{P}^3$, the equation of $\overline{X}$ becomes

$$
\begin{aligned}
&A(x^2t^2 + y^2z^2) + B(y^2t^2 + z^2x^2) + C(z^2t^2 + x^2y^2) + Dxyzt \\
&E(yt + zx)(zt + xy) + G(zt + xy)(xt + yz) + H(xt + yz)(yt + zx) \\
&= 0.
\end{aligned}
$$

There are 15 Göpel subgroups.

HW involutions

Restriction of the Cremona involution $\sigma_W : (s_i) \mapsto (s_i^{-1})$ of $\mathbb{P}^4$ to X_W, where W: a Weber hexad (definition omitted), X_W: another quartic model of X.

$$\overline{X} \overset{|\mathcal{O}_{\mathbb{P}^3}(2) - W|}{\dashrightarrow} X_W \subset \mathbb{P}^4.$$

Theorem[Hutchinson] The equation of X_W is

$$\sum_{i=1}^{5} s_i = \sum_{i=1}^{5} \lambda_i / s_i = 0, \quad \lambda_i \in \mathbb{C}^*.$$

We obtain 6 HW involutions.

4 Sketch of the Proof

We compute certain invariant, *the patching subgroups* of free involutions. For our X, it exactly classifies the isom. classes of quotient Enriques surfaces. The definition of it uses Nikulin's lattice theory.